OITS-533, UH-511-779-94

Gluon dipole penguin contributions to ϵ'/ϵ and CP violation in Hyperon decays in the Standard Model

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(Jan.. 1994)

Abstract

We consider the gluon dipole penguin operator contributions to ϵ'/ϵ and CP violation in hyperon decays. It has been proposed by Bertolini et al. that the contribution to ϵ'/ϵ may be significant. We show that there is a cancellation in the leading order contribution and this contribution is actually suppressed by a factor of order $O(m_\pi^2, m_K^2)/\Lambda^2$. We find that the same operator also contributes to CP violation in hyperon decays where it is not suppressed. The gluon dipole penguin operator can enhance CP violation in hyperon decays by as much as 25%.

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In this paper we study the gluon dipole penguin operator $\bar{s}\sigma^{\mu\nu}t^a G^a_{\mu\nu}(1-\gamma_5)d$ contributions to ϵ'/ϵ and CP violation in hyperon decays in the Standard Model (SM). The effective $\Delta S = 1$ Hamiltonian at leading order can be parametrized as

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_i C_i(\mu) Q_i(\mu) \ , \tag{1}$$

where the sum is over the effective operators $i = 1 - 10$ defined in Ref. [1], and the operators

$$\begin{aligned} Q_{11} &= \frac{g_s}{16\pi^2} m_s \bar{s}\sigma_{\mu\nu} t^a G^{\mu\nu}_a (1-\gamma_5) d \ , \\ Q_{12} &= \frac{eQ_d}{16\pi^2} m_s \bar{s}\sigma^{\mu\nu} F_{\mu\nu}(1-\gamma_5) d \ , \end{aligned} \tag{2}$$

where $G^{\mu\nu}_a$ and $F^{\mu\nu}$ are the gluon and photon field strengths, respectively. t^a is the $SU(3)_C$ generators normalized as $Tr(t^a t^b) = \delta^{ab}/2$. $C_i = z_i + y_i \tau$ with $\tau = -V_{td}V^*_{ts}/V_{ud}V^*_{us}$. CP violation is proportional to y_i. The QCD corrected coefficients y_i in the SM have been evaluated in Ref. [1] and Ref. [2]. In our later calculation we will use the values in Ref. [2] for y_i. The contributions to ϵ'/ϵ [1] and CP violation in hyperon decays [3–5] from the operators $Q_1 - Q_{10}$ have been extensively studied before. The contributions to ϵ'/ϵ from $Q_{11,12}$ have been considered recently by Bertolini et al. [2]. The dominant contributions come from the internal top quark. It was claimed that the contribution from Q_{11} is sizable, while the contribution from Q_{12} is negligible. In this paper we reconsider the Q_{11} contribution to ϵ'/ϵ and also consider the contribution to CP violation in hyperon decays. We find that there is a cancellation in the contribution to ϵ'/ϵ and the result obtained in Ref. [2] is strongly suppressed. There are substantial contributions to CP violation in some of the hyperon decays.

Contribution to ϵ'/ϵ

The parameter ϵ'/ϵ is a measure of CP violation in $K_{L,S} \to 2\pi$ decays. It is defined as

$$\frac{\epsilon'}{\epsilon} = i \frac{e^{i(\delta_2 - \delta_0)}}{\sqrt{2}(i\xi_0 + \bar{\epsilon})} \left| \frac{A_2}{A_0} \right| (\xi_2 - \xi_0) \ , \tag{3}$$

where $\bar{\epsilon} \approx 2.27 \times 10^{-3} e^{i\pi/4}$ is the CP violating parameter in $K^0 - \bar{K}^0$, δ_i are the strong rescattering phases, $|ReA_2/ReA_0| \approx 1/22$, and $\xi_i = ImA_i/ReA_i$. Here A_0 and A_2 are the

decay amplitudes with $I = 0$ and 2 in the final states, respectively. In order to evaluate the Q_{11} contribution to ϵ'/ϵ, one needs to calculate the matrix element $< \pi^0\pi^0|Q_{11}|\bar{K}^0 >$. Q_{11} is a $\Delta I = 1/2$ operator which contributes to ImA_0 only. If one uses the naive PCAC result as in Ref. [2], one obtains

$$< \pi^+\pi^-|Q_{11}|\bar{K}^0 >= \frac{g_s m_s}{16\pi^2 f_\pi^2 f_K}\left[< 0|\bar{d}\sigma_{\mu\nu}t^a G_a^{\mu\nu} d|0 > + < 0|\bar{s}\sigma_{\mu\nu}t^a G_a^{\mu\nu} s|0 >\right] , \tag{4}$$

where $f_\pi = 132MeV$ and $f_K = 161MeV$. The matrix element on the right-hand side of eq.(3) can be related to the quark condensation by

$$g_s < 0|\bar{q}\sigma_{\mu\nu}t^a G_a^{\mu\nu} q|0 >= m_0^2 < 0|\bar{q}q|0 > , \tag{5}$$

where $m_0^2 \approx 1\ GeV^2$ [6] is a phenomenological constant. Using

$$< 0|\bar{u}u + \bar{s}s|0 >= -\frac{f_K^2 m_K^2}{m_u + m_s} , \tag{6}$$

and $< 0|\bar{d}d|0 >=< 0|\bar{u}u|0 >$, one finally obtains

$$< \pi^+\pi^-|Q_{11}|\bar{K}^0 >= -\frac{m_0^2 m_s}{16\pi^2(m_u + m_s)}\frac{m_K^2 f_K}{f_\pi^2} . \tag{7}$$

One also has, $< \pi^0\pi^0|Q_{11}|\bar{K}^0 >=< \pi^+\pi^-|Q_{11}|\bar{K}^0 >$.

Using the matrix element in eq.(7), a quite large contribution to ϵ'/ϵ was obtained in Ref. [2]. They find that Q_{11} contributes between $(2 \sim 3) \times 10^{-4}$. It weakly depends on the top quark mass m_t for m_t between 100 GeV to 250 GeV. We would like to point out that the naive PCAC result obtained above is not correct. The calculation in Ref. [2] is only part (Fig. 1.a) of the contributions. An important "tadpole" contribution (Fig. 1.b) was not considered in the analysis of Ref. [2]. This contribution cancels exactly the PCAC result obtained above. The net contribution is much smaller. The situation is the same as for ϵ'/ϵ in the Weinberg model of CP violation [7,8]. The importance of the "tadpole" contribution was first noticed by Donoghue and Holstein [7] in the Weinberg model of CP violation. We now calculate these contributions in the SM.

The leading order chiral realization of Q_{11} for $\bar{K}^0 \to n\pi$ is

$$L = aTr(\lambda_6 + i\lambda_7)U) + H.C.\ . \tag{8}$$

The conventional parametrization of U is given by $U = exp(-i4t^a\phi^a/f_\pi)$ with ϕ^a being the fields of the pseudoscalars. In our discussion we follow Ref. [8] using a more general parametrization of U. To fourth power in the meson fields, there is a free parameter a_3 [8,9],

$$U = 1 + i\frac{2}{f_\pi}(2t^a\phi^a) - \frac{2}{f_\pi^2}(2t^a\phi^a)^2 - i\frac{a_3}{f_\pi^3}(2t^a\phi^a)^3 + \frac{2(a_3-1)}{f_\pi^4}(2t^a\phi^a)^4 + ... \tag{9}$$

The conventional parametrization corresponds to $a_3 = 4/3$. The on-shell amplitudes should not depend on a_3. The effective lagrangian in eq.(8) will not only generate the direct $K \to 2\pi$ amplitude (Fig. 1.a), but also generate a non-zero $K \to vaccum$ transition amplitude. This non-zero $K \to vaccum$ transition amplitude, when combined with the strong $K\pi\bar{K}\pi$ amplitude $A_{strong}(K\bar{K}\pi^0\pi^0)$ with one K off-shell with $q^2 = 0$, will also generate a $K \to 2\pi$ amplitude as show in Fig. 1.b. The total amplitude for $K \to 2\pi$ from Q_{11} is the sum of contributions from Fig. 1.a and Fig. 1.b. We have

$$\begin{aligned} &A_{\ total}(\bar{K}^0 \to \pi^0\pi^0) = A(fig.1.a) + A(fig.1.b) \\ &= A(\bar{K}^0 \to \pi^0\pi^0)|_{fig.1.a} + A_{strong}(K^0\bar{K}^0\pi^0\pi^0)\frac{1}{m_K^2}A(\bar{K}^0 \to vaccum)\ , \end{aligned} \tag{10}$$

where $A(\bar{K}^0 \to n\pi) =< n\pi| - H_{eff}(Q_{11})|\bar{K}^0 >$, and $H_{eff}(Q_{11}) = -i\frac{G_F}{\sqrt{2}}y_{11}Im(V_{td}V_{ts}^*)Q_{11}$. We have

$$\begin{aligned} A(\bar{K}^0 \to vaccum) &= -i\frac{G_F}{\sqrt{2}}y_{11}Im(V_{td}V_{ts}^*)f_\pi\sqrt{2}\frac{g_s m_s}{32\pi^2}\tilde{A}_{\bar{K}\pi}\ , \\ A(\bar{K}^0 \to \pi^0) &= -\frac{G_F}{\sqrt{2}}y_{11}Im(V_{td}V_{ts}^*)\frac{g_s m_s}{32\pi^2}\tilde{A}_{\bar{K}\pi}\ , \\ A(\bar{K}^0 \to \pi^0\pi^0) &= i\frac{G_F}{\sqrt{2}}y_{11}Im(V_{td}V_{ts}^*)\frac{g_s m_s}{32\sqrt{2}\pi^2 f_\pi}\tilde{A}_{K\pi}\frac{a_3}{2}\ . \end{aligned} \tag{11}$$

Here $\tilde{A}_{K\pi} = -i < \pi^0|\bar{s}\sigma^{\mu\nu}2t^aG^a_{\mu\nu}(1-\gamma_5)d|\bar{K}^0 >$. Note that $A(\bar{K}^0 \to \pi^0\pi^0)$ is a_3 dependent, which can not be the final answer. Additional contribution from Fig. 1.b has to be considered. What has been evaluated in Ref. [2] conrresponds to $A(fig.1.a)$ with $a_3 = 2$. $m_s\tilde{A}_{K\pi}$ has been calculated to be $0.11 \sim 0.17\ GeV^4$ in the MIT bag model [10]. Using this value one obtains approximately the same numerical value for $A(fig.a)$ as obtained in Ref. [2].

To calculate the contribution from Fig. 1.b, one needs to know the strong $K\pi\bar{K}\pi$ amplitude A_{strong}. This can be obtained from the leading chiral largrangian

$$L = \frac{f_\pi^2}{8}[Tr\partial_\mu U\partial^\mu U + BTr(MU + U^\dagger M)] \ , \tag{12}$$

where $M = Diag(m_u \ , m_d \ , m_s)$ is the quark mass matrix and B is a constant. From this we obtain [8,9]

$$A_{strong}(K^0\bar{K}^0\pi^0\pi^0) = \frac{m_K^2}{2f_\pi^2}\frac{a_3}{2} \ . \tag{13}$$

Here we have set the momentum of K annihilated into the vacuum to be zero and others to be on-shell (Of course the on-shell A_{stong} is a_3 independent.). Inserting eq.(13) into eq.(10), we find the a_3 dependent contribution from Fig. 1.b cancels exactly that from Fig. 1.a. To this order, there is no contribution to ϵ'/ϵ from Q_{11}. However, the cancellation may not be complete due to higher derivative terms in chiral perturbation theory. Unfortunately, one does not know how to calculate higher-order contributions to the matrix elements. One can define a suppression factor D,

$$A_{total}(K^0 \to \pi^0\pi^0) = A(fig.1.a)D \ . \tag{14}$$

The suppression factor should be of order $O(p^2)/\Lambda^2 \approx O(m_K^2, m_\pi^2)/\Lambda^2$ [7,8]. Here $\Lambda \approx 1\ GeV$ is the chiral symmetry breaking scale. The contribution is suppressed by a factor of 0.3 or even more. The contribution from Q_{11} calculated in Ref. [2] correspond to $D = 1$.

We list the results for $D = m_K^2/\Lambda^2$ in Table 1, but note that the sign of the contribution is unknown. The parameter $Im(V_{td}V_{ts}^*)$ is constrained by the parameter ϵ from $K^0 - \bar{K}^0$ mixing. Unfortunately it is not completely fixed. It can vary between 3×10^{-4} to 10^{-4} for top quark mass m_t about 100 GeV and 2×10^{-4} to 0.5×10^{-4} for m_t about 200 GeV [11]. In Table 1, following Ref. [2], we use an approximation $Im(V_{td}V_{ts}^*) = 2.77 \times 10^{-4}(m_t^2/m_W^2)^{-0.365}$ as the central value. We see that the effect of Q_{11} is generally insignificant, but becomes important when the contribution from $Q_1 - Q_{10}$ become small due to cancellations for m_t around 200 GeV.

CP violation in hyperon decays

CP violation in hyperon decays in the SM has been studied before [3–5]. The Q_{11} contributions were not included, and we now turn to study these contributions. Non-leptonic hyperon decays proceed into both S-wave (parity-violating) and P-wave (parity-conserving) final states with amplitudes S and P, respectively. One can write the amplitude in the rest frame of the initial baryon as

$$Amp(B_i \to B_f \pi) = S + P\vec{\sigma} \cdot \vec{q}\,, \tag{15}$$

where $\vec{q}$ is the momentum of pion. It is convenient to write the amplitudes as

$$\begin{aligned} S &= \sum_i S_i e^{i(\phi_i^S + \delta_i^S)} \\ P &= \sum_i P_i e^{i(\phi_i^P + \delta_i^P)} \end{aligned} \tag{16}$$

to explicitly separate the strong rescatering phases δ_i and the weak CP violating phases ϕ_i. In the rest frame of the initial baryon, one particularly interesting observable is the asymmetry A defined in Ref. [4]

$$A = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}\,, \tag{17}$$

where $\alpha = 2Re(S^* P)/(|S|^2 + |P|^2)$, and $\bar{\alpha}$ is the corresponding quantity for anti-hyperon decays. A non-zero A signals CP violation. In this paper we will concentrate on the study of A. The results can be easily generalized to other CP violating observables defined in Ref. [4]. We will calculate the CP violating observable A for $\Lambda \to N\pi$ and $\Xi \to \Lambda\pi$. Since to leading order the observables $A(\Lambda^0_-)$ and $A(\Xi^-_-)$ for $\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$ are the same as $A(\Lambda^0_0)$ and $A(\Xi^0_0)$ for $\Lambda \to n\pi^0$ and $\Xi^0 \to \Lambda\pi^0$, respectively, we choose to work with $\Lambda \to n\pi^0$ and $\Xi^0 \to \Lambda\pi^0$ decays.

For the same reason as for the $K \to 2\pi$ amplitude, when evaluating the S-wave amplitude, one should also include the contributions from the direct contribution of Fig. 2.a and the "tadpole" contribution of Fig. 2.b. We have [4,12]

$$
\begin{aligned}
S(\Lambda\to n\pi^0) &= -\frac{i}{\sqrt{2}f_\pi} < n|H^\dagger_{eff}(Q_{11})|\Lambda > |_{fig.2.a} \\
&+ \frac{i}{\sqrt{2}f_\pi}\left[\frac{3}{2}\right]^{1/2}\frac{M_\Lambda - M_n}{m_s - m_d}\left[\frac{m_s+m_d}{if_K M_K^2}\sqrt{2} < 0|H^\dagger_{eff}(Q_{11})|K^0 >\right]_{fig.2.b} ,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
S(\Xi^0\to\Lambda\pi^0) &= -\frac{i}{\sqrt{2}f_\pi} < \Lambda|H^\dagger_{eff}(Q_{11})|\Xi^0 > |_{fig.2.a} \\
&- \frac{i}{\sqrt{2}f_\pi}\left[\frac{3}{2}\right]^{1/2}\frac{M_\Xi - M_\Lambda}{m_s - m_d}\left[\frac{m_s+m_d}{if_K M_K^2}\sqrt{2} < 0|H^\dagger_{eff}(Q_{11})|K^0 >\right]_{fig.2.b} .
\end{aligned}
\tag{19}
$$

We use pole model to calculate the P-wave amplitudes. For consistency, one should include both the baryon and Kaon poles [12]. The "tadpole" contribution from Fig. 2.b in the S-wave amplitude and the Kaon pole contribution in the P-wave amplitude are both sizable. However, the CP violating observable A depends on the difference between the S-wave phase ϕ^S and the P-wave phase ϕ^P. There is substantial cancellation between the "tadpole" contribution in the S-wave and the Kaon pole contribution in the P-wave.

The contributions to CP violation in hyperon decays from the same operator Q_{11} have been studied in the Weinberg model [3,4]. We can use some of the results from there. However in the Weinberg model the CP violating parameter $\bar\epsilon$ in the $K^0-\bar K^0$ is also generated by the operator Q_{11} through long distance π, η and η' pole contributions, the coefficient $C_{11}(Weinberg)$ of Q_{11} [7] is fixed. In our case, the contribution to $\bar\epsilon$ from Q_{11} is small. To obtain the CP violating phases in the decay amplitudes, we can repeat the MIT bag model calculations in Ref. [4]. In fact we can obtain the phases by replacing the matrix element $< \pi^0| -\frac{G_F}{\sqrt{2}}Im(V^*_{ud}V_{us}C^*_{11}(Weinberg))Q^\dagger_{11}|K^0) >$ in the Weinberg model by the SM matrix element, $< \pi^0| - H^\dagger_{eff}(Q_{11})|K^0 >$, and rescale the phases. We define the rescaling factor R as

$$
\begin{aligned}
R &= \frac{< \pi^0| - H^\dagger_{eff}(Q_{11})|K^0 >}{< \pi^0| -\frac{G_F}{\sqrt{2}}Im(V^*_{ud}V_{us}C^*_{11}(Weinberg))Q^\dagger_{11}|K^0) >} \\
&= -\frac{G_F}{\sqrt{2}}y_{11}Im(V_{td}V^*_{ts})\frac{g_s}{32\pi^2}\frac{m_s\tilde A_{K\pi}}{5.8\times10^{-11}\ GeV^2} .
\end{aligned}
\tag{20}
$$

Here we have used $< \pi^0| -\frac{G_F}{\sqrt{2}}Im(V^*_{ud}V_{us}C^*_{11}(Weinberg))Q^\dagger_{11}|K^0) >= 5.8\times10^{-11}\ GeV^2$ [4]. We will follow Weinberg to use $g_s\approx 4\pi/\sqrt{6}$ [13]. The CP violating phases in hyperon decays

from Q_{11} in the SM are now obtained by multiplying R to the phases obtained in Ref. [4] for the Weinberg model.

In Table 2 we list the CP violating observables A for Λ and Ξ decays from different operators. From Table 2, we see that the Q_{11} contribution to CP violation in $\Lambda \to N\pi$ is negligible. However the contribution to CP violation in $\Xi \to \Lambda\pi$ can be important. The result is to enhance CP violation in Ξ decays by about 17%. If we use the upper value of $0.17\ GeV^4$ for $m_s\tilde{A}_{K\pi}$, the enhancement will be 25%. The allowed regions for $A(\Lambda)$ and $A(\Xi)$ without the contributions from Q_{11} are $3 \times 10^{-5} \sim 0.4 \times 10^{-5}$ and $5.3 \times 10^{-5} \sim 0.7 \times 10^{-5}$. With the new contributions from Q_{11} the allowed region for $A(\Lambda)$ is not changed, but the allowed region for $A(\Xi)$ becomes to $6.6 \times 10^{-5} \sim 0.7 \times 10^{-5}$. This may be tested in the future [14].

ACKNOWLEDGMENTS

This work was supported in part by the Department of Energy Grant No. DE-FG06-85ER40224 and DE-AN03-76SF00235. We thank E. Golowich for reading the manuscript.

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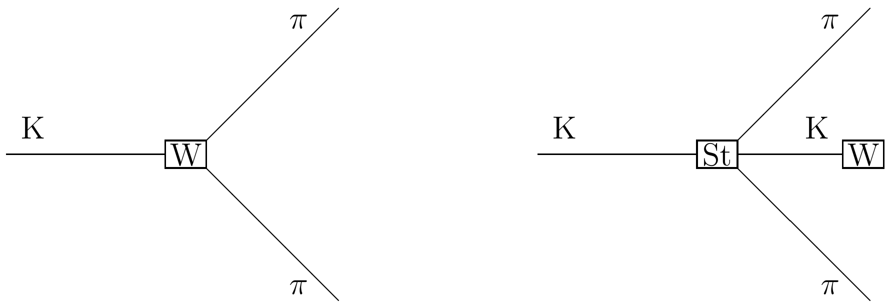
TABLES

TABLE I. $\epsilon'/\epsilon \times 10^4$ for $Im(V_{td}V_{ts}^*) = 2.77 \times 10^{-4}(m_t^2/m_W^2)^{-0.365}$.

m_t(GeV)	130	170	200	230
$\Lambda_4 = 200\ MeV$				
$Q_1 - Q_{10}$	6.3	2.8	0.7	-1.4
Q_{11}	0.65	0.58	0.5	0.48
$\Lambda_4 = 300\ MeV$				
$Q_1 - Q_{10}$	7.8	3.6	1.0	-1.7
Q_{11}	0.73	0.63	0.55	0.50

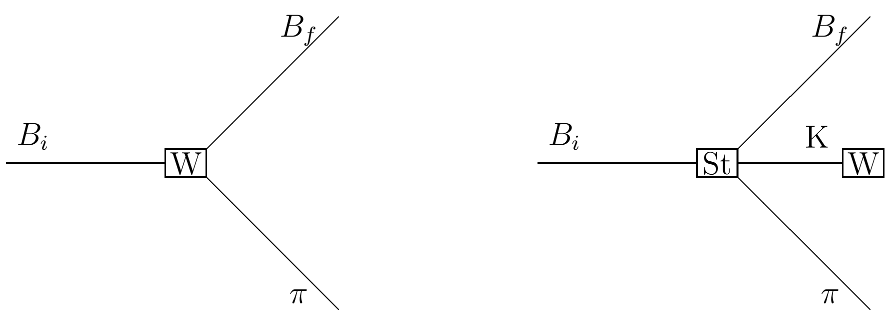
TABLE II. $A \times 10^5$ for $Im(V_{td}V_{ts}^*) = 2.77 \times 10^{-4}(m_t^2/m_W^2)^{-0.365}$ and $m_s\tilde{A}_{K\pi} = 0.12\ GeV^4$.

m_t(GeV)	130	170	200	230
$\Lambda_4 = 200\ MeV$				
$A(\Lambda^0_-)(Q_1 - Q_{10})$	-1.5	-1.3	-1.1	-1.0
$A(\Lambda^0_-)(Q_{11})$	-0.043	-0.037	-0.033	-0.031
$A(\Xi^-_-)(Q_1 - Q_{10})$	-2.6	-2.2	-1.9	-1.8
$A(\Xi^-_-)(Q_{11})$	-0.55	-0.47	-0.41	-0.40
$\Lambda_4 = 300\ MeV$				
$A(\Lambda^0_-)(Q_1 - Q_{10})$	-2.0	-1.7	-1.5	-1.4
$A(\Lambda^0_-)(Q_{11})$	-0.046	-0.039	-0.034	-0.032
$A(\Xi^-_-)(Q_1 - Q_{10})$	-3.4	-2.9	-2.5	-2.3
$A(\Xi^-_-)(Q_{11})$	-0.58	-0.50	-0.44	-0.40



1.a 1.b

Fig. 1. Contributions to $K \to \pi\pi$ amplitude. Here W and St indicate weak and strong interactions, respectively.



2.a 2.b

Fig. 2. Contributions to the S-wave $B_i \to B_f\pi$ amplitudes.